

Remark about scaling limit of ABJ theory

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Remark about scaling limit of ABJ theory

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ABSTRACT: We generalize the suggestion presented in arXiv:0806.3498 that the $3d$ $N = 8$ superconformal $SU(N)$ Chern-Simons-matter theory of Lorentzian Bagger-Lambert- Gustavson type (L-BLG) can be obtained through the scaling limit from $N = 6$ superconformal $U(N) \times U(N)$ Chern-Simons-matter theory of Aharony, Bergman, Jafferis and Maldacena (ABJM) to the case when we study the scaling limit of $N = 6$ superconformal $U(M) \times U(N)$ Chern-Simons-matter theory of Aharony, Bergman and Jafferis (ABJ). We show that if we extend the ABJ theory in the same way as in arXiv:0811.1540 we can define a consistent limit that leads to $SU(N)$ L-BLG theory together with $U(M - N)_k$ Chern-Simons theory of level k .

KEYWORDS: AdS-CFT Correspondence, M-Theory

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1 Introduction and summary

It is a long standing problem to find a world-volume theory of N M2-branes that can be considered as a generalization of the world-volume theory of a single M2-brane [1, 2].¹ Following very nice analysis performed in [6], Bagger and Lambert and Gustavson (BLG) formulated a three-dimensional superconformal Chern-Simons-matter theory that successfully captures many aspects of dynamics of N M2-branes [7–9]. The remarkable property of this theory that it makes exceptional is that is based on 3–algebra. On the other hand it was soon discovered that the original formulation of BLG theory describes only two coincident M2-branes on condition that the 3–algebra is kept finite and it has a positive-definite metric [11–17].

In order to resolve this limitation it was suggested in [17–21] to use 3–algebra with a Lorentzian (indefinite) signature metric. The resulting Lorentzian-BLG (L-BLG) theory is $N = 8$ superconformal at the classical level even if its interpretation as a quantum field theory is still an open problem. In particular, if one expand near a classical vacuum that spontaneously breaks the superconformal symmetry it becomes equivalent [22, 23, 25] to a standard low-energy gauge theory of multiple D2-branes that is non-conformal $N = 8$ supersymmetric $3d$ $N = 8$ SYM theory.

A different $3d$ superconformal Chern-Simons-matter theory was proposed by Aharony, Bergman, Jafferis and Maldacena (ABJM) [26]. This theory possesses $N = 6$ supersymmetry [27] and it is interpreted as a theory that describes N coincident M2-branes at the singularity of the orbifold $\mathbf{C}^4/\mathbf{Z}_k$. While the ABJM theory also admits a 3–algebra interpretation [28] it seems to be different from the original L-BLG theory. More precisely, these theories have different field content and different symmetries.

An interesting suggestion how these two theories are related was presented in [29] (see also [30]), where it was argued that the L-BLG theory can be interpreted as a certain limit of the ABJM theory in which one sends the ABJM coupling k (CS level) to infinity and at the same time rescales some of the fields to zero so that they decouple.

¹For review and extensive list of references considering early years of M2-brane theories, see [3–5].

This proposal was further clarified in [31] where it was argued that in order to relate these two theories by scaling limit we have to supplement the ABJM theory with an extra ghost multiplet that is decoupled from the ABJM fields. Further it was explicitly shown that there exists a limit of the 3–algebra of ABJM theory that is trivially extended by an extra ghost generator that leads to the Lorentzian 3–algebra of L-BLG theory. It was also suggested there that it is possible to interpret this scaling limit as a definition of L-BLG theory in terms of ABJM theory. In particular, using this limit it can be seen the relation between the L-BLG theory and the 3d $N = 8$ SYM theory that describes N D2-branes: Taking the scaling limit and then giving one of the scalars an expectation value is equivalent to the procedure [23, 24] for obtaining the D2-brane theory from the ABJM theory.

Further more general forms of ABJM theory were introduced in [32]. One example of such a more general theory is $U(M)_k \times U(N)_{-k}$ Chern-Simons-matter that has the same matter content and interactions as in [26] but with $M \neq N$. From the point of view of M2-branes as probes of $\mathbf{C}^4/\mathbf{Z}_k$ singularity these theories arise (for $M > N$) when we have $(M - N)$ fractional M2-branes that are localized at the singularity together with N M2-branes that are free to move around. It was argued in [32] that these theories exist as well defined quantum field theories in case when $|M - N| \leq k$ and that in this case the gravitational dual description is $AdS_4 \times \mathbf{S}^7/\mathbf{Z}_k$ background as in [26] but with an additional "torsion flux" that takes values in $H^4(\mathbf{S}^7/\mathbf{Z}_k, \mathbf{Z}) = \mathbf{Z}_k$.

Since the ABJ theory [26] is more general than the ABJM theory we mean that it deserves to be studied further. In particular, we would like to see how to define the scaling limit similar to the limit given in [29] in case of the ABJ theory. In this note we introduce such a scaling limit and show that it leads to well defined theory. Concretely, we implement such a scaling limit that corresponds to the decoupling of all massive degrees of freedom when we move N M2-branes far away from the origin of $\mathbf{C}^4/\mathbf{Z}_k$. According to general arguments [26] we can expect that the low energy modes are pure $U(M - N)_k$ Chern-Simons theory that describes dynamics of $N - M$ fractional M2-branes localized at the origin of $\mathbf{C}^4/\mathbf{Z}_k$ together with L-BLG theory that describes dynamics of N M2-branes in flat space. In fact, we will study the spectrum of fluctuation modes around the vacuum state that corresponds to N M2-branes moving from the origin and we identify spectrum of massless and massive modes that agrees with observation given in [26]. Then we define such a scaling limit that decouples these massive modes and retains the massless ones together with auxiliary fields that are well known from L-BLG theory. We explicitly show, following [29] and [31] that this limit leads to $U(N - M)_k$ CS theory together with L-BLG theory and that these two theories are decoupled.

This result implies that L-BLG theory can be defined from ABJ theory as well in the limit when we appropriately redefine the fields and the level k , add ghosts fields [31] and then send the small parameter to zero.

The extension of this work is as follows. We can ask the question how to define the decoupling limit and what is the resulting theory in case of $(U(N) \times U(N))^n$ superconformal quiver gauge theories [27] (see also [33–36]). We hope to return to this problem in future.

The organization of this paper is as follows. In the next section (2) we review basic facts considering ABJ theory. Then in section (3) we analyze the situation when N M2-branes

is localized far away from the origin of $\mathbf{C}^4/\mathbf{Z}_k$. We determine massless and massive modes that propagate around this vacuum solution. Using this result we introduce in section (4) the scaling limit in ABJ theory that decouple the massive and massless modes and leads to $U(N - M)_k$ CS theory of level k together with $SU(N)$ L-BLG theory.

2 Aspect of ABJ theory

The difference between ABJM and ABJ theory is that fractional M2-branes are added as a new parameter in dual theory. In other words theory possesses three parameters M, N, k that all are integer valued. Now we briefly review basic facts considering this theory

- Let us consider M2-brane as a probe of $\mathbf{C}^4/\mathbf{Z}_k$ singularity. As was argued in [32] the ABJ theories arise when, in addition to N M2-branes that can move around $\mathbf{C}^4/\mathbf{Z}_k$ singularity, $(N - M)$ fractional M2-branes that are localized at orbifold singularity.
- The classical field theory description of this theory is given by $N = 6$ superconformal theory with gauge group $U(M) \times U(N)$ where $M \neq N$. On the other hand it was argued in [32] that these theories exist as a unitary superconformal theories only for $|M - N| \leq k$.
- The gravity dual of these unitary theories is $AdS_4 \times \mathbf{S}^7/\mathbf{Z}_k$ background that was originally introduced in [26] but now with additional "torsion flux" that takes values in $H^4(\mathbf{S}^7/\mathbf{Z}_k, \mathbf{Z}) = \mathbf{Z}_k$. In fact, this is a good description of the gravitational dual when $N \gg k^5$. In case when $k \ll N \ll k^5$ the appropriate description is in terms of type IIA string theory on $AdS_4 \times CP^3$ with a discrete holonomy of the NSNS 2-form field in the $CP^1 \subset CP^3$.

Let us be more concrete in description of ABJ theory. This theory is $N = 6$ supersymmetric Chern-Simons-matter theory with two gauge groups or ranks M, N and levels k and $-k$ respectively. Further, this theory is characterized by following properties:

- Gauge and global symmetries:

$$\text{gauge symmetry: } U(M) \otimes U(N)$$

$$\text{global symmetry: } SU(4)$$

- The field content of given theory is as follows:

$$A_\mu^{(L)} : \text{Adj}(U(M)) , \quad A_\mu^{(R)} : \text{Adj}(U(N)) . \tag{2.1}$$

Further we have $M \times N$ matrix valued matter fields-4 complex scalar $Y^A (A = 1, 2, 3, 4)$ and their hermitian conjugates Y_A^\dagger . We have also $M \times N$ matrix valued fermions ψ_A together with their hermitian conjugates $\psi^{A\dagger}$. Fields with raised A index transform in the $\mathbf{4}$ of R symmetry $SU(4)$ group and those with lowered index transform in the $\bar{\mathbf{4}}$ representations.

The corresponding Lagrangian has the following form

$$\begin{aligned}
 \mathcal{L} = & -\text{Tr} \left(D^\mu Y_A^\dagger D_\mu Y^A \right) - i \text{Tr} \left(\bar{\psi}^{A\dagger} \gamma^\mu D_\mu \psi_A \right) - V + \mathcal{L}_{CS} - \\
 & -i \frac{2\pi}{k} \text{Tr} \left(\bar{\psi}^{A\dagger} \psi_A Y_B^\dagger Y^B - \bar{\psi}^{A\dagger} Y^B Y_B^\dagger \psi_A \right) + 2 \frac{2\pi}{k} \text{Tr} \left(\bar{\psi}^{A\dagger} \psi_B Y_A^\dagger Y^B - \bar{\psi}^{A\dagger} Y^B Y_A^\dagger \psi_B \right) + \\
 & + i \frac{2\pi}{k} \epsilon_{ABCD} \text{Tr} \left(\bar{\psi}^{A\dagger} Y^C \psi^{B\dagger} Y^D \right) - i \frac{2\pi}{k} \epsilon^{ABCD} \text{Tr} \left(Y_D^\dagger \bar{\psi}_A Y_C^\dagger \psi_B \right), \quad (2.2)
 \end{aligned}$$

where \mathcal{L}_{CS} is a Chern-Simons term and $V(Y)$ is a sextic scalar potential

$$\begin{aligned}
 \mathcal{L}_{CS} = & \frac{k}{4\pi} \epsilon^{\mu\nu\lambda} \text{Tr} \left[A_\mu^{(L)} \partial_\nu A_\lambda^{(L)} + \frac{2i}{3} A_\mu^{(L)} A_\nu^{(L)} A_\lambda^{(L)} \right] - \\
 & - \frac{k}{4\pi} \epsilon^{\mu\nu\lambda} \text{Tr} \left[A_\mu^{(R)} \partial_\nu A_\lambda^{(R)} + \frac{2i}{3} A_\mu^{(R)} A_\nu^{(R)} A_\lambda^{(R)} \right], \\
 V(Y) = & - \frac{4\pi^2}{3k^2} \text{Tr} \left[Y^A Y_A^\dagger Y^B Y_B^\dagger Y^C Y_C^\dagger + Y_A^\dagger Y^A Y_B^\dagger Y^B Y_C^\dagger Y^C + \right. \\
 & \left. + 4 Y^A Y_B^\dagger Y^C Y_A^\dagger Y^B Y_C^\dagger - 6 Y^A Y_B^\dagger Y^B Y_A^\dagger Y^C Y_C^\dagger \right]. \quad (2.3)
 \end{aligned}$$

Further, the covariant derivatives are defined as

$$D_\mu Y^A = \partial_\mu Y^A + i A_\mu^{(L)} Y^A - i Y^A A_\mu^{(R)}, \quad D_\mu Y_A^\dagger = \partial_\mu Y_A^\dagger - i Y_A^\dagger A_\mu^{(L)} + i A_\mu^{(R)} Y_A^\dagger \quad (2.4)$$

and for fermions

$$D_\mu \psi_A = \partial_\mu \psi_A + i A_\mu^{(L)} \psi_A - i \psi_A A_\mu^{(R)}, \quad D_\mu \psi^{A\dagger} = \partial_\mu \psi^{A\dagger} - i \psi^{A\dagger} A_\mu^{(L)} + i A_\mu^{(R)} \psi^{A\dagger}. \quad (2.5)$$

After review of basic properties of ABJ theory we will analyze the spectrum of fluctuation modes around the configuration when N M2-branes are localized far away from the origin of $\mathbf{C}^4/\mathbf{Z}_k$.

3 N M2-branes displaced from the origin of $\mathbf{C}^4/\mathbf{Z}_k$

In this section we study solutions of the $U(M) \times U(N)$ theory that describes situation when we move N M2-branes from the origin of $\mathbf{C}^4/\mathbf{Z}_k$. For concreteness we presume that $M > N$. Then it is natural to write $A_\mu^{(L)}$ in the form

$$A_\mu^{(L)} = \begin{pmatrix} A_{11\mu}^{(L)} & A_{12\mu}^{(L)} \\ A_{21\mu}^{(L)} & A_{22\mu}^{(L)} \end{pmatrix}, \quad (3.1)$$

where $A_{11\mu}^{(L)}$ is $(N-M) \times (N-M)$ matrix, $A_{12\mu}^{(L)}$ are $(M-N) \times N$ and $A_{21\mu}^{(L)}$ $N \times (M-N)$ matrices. Finally $A_{22\mu}^{(L)}$ is $N \times N$ matrix. In the same way we write

$$Y^A = \begin{pmatrix} Z^A \\ Y_0^A \mathbf{I}_{N \times N} + \tilde{Y}^A \end{pmatrix} \quad (3.2)$$

where Z^A is $(M-N) \times N$ matrix and \tilde{Y}^A is $N \times N$ matrix with $\text{Tr} \tilde{Y}^A = 0$.

We are interested in configuration when we separate N M2-branes far away from the origin of $\mathbf{C}^4/\mathbf{Z}_k$. In other words we consider the solution of the equation of motion of ABJ theory in the form

$$Y^A = \begin{pmatrix} 0_{M \times N} \\ R^A \mathbf{I}_{N \times N} \end{pmatrix} \quad (3.3)$$

and where all other fields are equal to zero. Then in order to find the spectrum of fluctuations we expand the scalar fields around the ansatz (3.3) as

$$Y^A = \begin{pmatrix} Z^A \\ (R^A + Y_0^A) \mathbf{I}_{N \times N} + \tilde{Y}^A \end{pmatrix}, \quad \text{Tr} \tilde{Y}^A = 0. \quad (3.4)$$

Inserting this ansatz into the definition of the covariant derivative (2.4) we obtain

$$\begin{aligned} & \partial_\mu Y^A + i A_\mu^{(L)} Y^A - i Y^A A_\mu^{(R)} = \\ & = \begin{pmatrix} \partial_\mu Z^A + i A_{11\mu}^{(L)} Z^A - i Z^A A_\mu - i \frac{1}{2} Z^A B_\mu + i A_{12\mu}^{(L)} (Y_0^A \mathbf{I}_{N \times N} + \tilde{Y}^A) \\ \partial_\mu Y_0^A \mathbf{I}_{N \times N} + \partial_\mu \tilde{Y}^A + i [A_\mu, \tilde{Y}^A] - i Y_0^A B_\mu - i \frac{1}{2} \{B_\mu, \tilde{Y}^A\} \end{pmatrix} + \begin{pmatrix} i A_{12\mu}^{(L)} R \\ -i B_\mu R \end{pmatrix}, \end{aligned} \quad (3.5)$$

where we introduced fields B_μ and A_μ as a combinations of $A_{22\mu}^{(L)}$ and A_μ^R

$$A_{22\mu}^{(L)} = A_\mu - \frac{1}{2} B_\mu, \quad A_\mu^R = A_\mu + \frac{1}{2} B_\mu \quad (3.6)$$

This result implies that due to the Higgs mechanism fields $A_{12\mu}^{(L)}$ and B_μ become massive with mass terms equal to

$$\text{Tr} \left(A_{12\mu}^{(L)} A_{21}^{(L)\mu} \right) R^2 = \text{Tr} \left(A_{12\mu}^{(L)} \left(A^{(L)\mu} \right)_{12}^\dagger \right) R^2, \quad \text{Tr} (B_\mu B^\mu) R^2, \quad (3.7)$$

where $R^2 = R_A R^A$ and where we used the fact that $(A_{12\mu}^{(L)})^\dagger = A_{21\mu}^{(L)}$. We see that these fields become infinite heavy in the limit $R \rightarrow \infty$. However there is an important difference between B_μ and $A_{12\mu}^{(L)}$ since B_μ is auxiliary field while $A_{12\mu}^{(L)}$ is massive vector field with ordinary kinetic term. To see this note that introducing the variables (3.6) the Chern-Simons Lagrangian density (2.3) can be rewritten into the form

$$\begin{aligned} \mathcal{L}_{CS} = & \frac{k}{2\pi} \epsilon^{\mu\nu\lambda} \text{Tr} \left[A_{11\mu}^{(L)} \partial_\nu A_{11\lambda}^{(L)} + \frac{2i}{3} A_{11\mu}^{(L)} A_{11\nu}^{(L)} A_{11\lambda}^{(L)} + \right. \\ & + 2 A_{12\mu}^{(L)} \partial_\nu A_{21\lambda}^{(L)} + 2i (A_{11\mu}^{(L)} A_{12\nu}^{(L)} A_{21\lambda}^{(L)} + A_{22\mu}^{(L)} A_{21\nu}^{(L)} A_{12\lambda}^{(L)}) - \\ & \left. - B_\mu (\partial_\nu A_\lambda - \partial_\lambda A_\nu + i [A_\nu, A_\lambda]) - \frac{i}{6} B_\mu B_\nu B_\lambda \right] \end{aligned} \quad (3.8)$$

that shows that there is no kinetic terms for B_μ that confirms the claim that B_μ is auxiliary.

Let us now consider the fluctuation modes Z^A, \tilde{Y}^A and Y_0^A . It is easy to see from the form of the scalar potential V_B that the fields Z^A are massive with mass proportional R^4 while Y_0^A, \tilde{Y}^A are massless.

In the same way we can proceed in case of fermions. Explicitly, we write bi-fundamental fermions as

$$\psi_A = \begin{pmatrix} \chi_A \\ \psi_A^0 \mathbf{I}_{N \times N} + \tilde{\psi}_A \end{pmatrix}, \quad \text{Tr} \tilde{\psi}_A = 0. \quad (3.9)$$

Then we can straightforwardly analyze the scalar part of the Lagrangian (2.2) and determine that the modes χ_A become massive with mass proportional to R^2 while the fields ψ_A^0 and $\tilde{\psi}_A$ are massless.

Let us now summarize the field content around the configuration with large $\langle Y^A Y_A^\dagger \rangle = R^2$ corresponding separation of N M2-branes from the origin of $\mathbf{C}^4/\mathbf{Z}_k$:

$$\begin{aligned} A_{11\mu}^{(L)}, \quad A_\mu, \quad \psi_A^0, \quad \tilde{\psi}_A, \quad Y_0^A, \quad \tilde{Y}^A &: \text{ massless,} \\ Z^A, \quad A_{12\mu}^{(L)}, \quad \chi_A &: \text{ massive,} \\ B_\mu &: \text{ auxiliary.} \end{aligned} \quad (3.10)$$

Since the scaling limit defined in [29] can be interpreted as a limit when we move the M2-branes infinitely far from the singularity together with sending k to infinity we can expect that similar limit exists in the ABJ theory as well.

4 Scaling limit

Motivated by the analysis performed in previous section we would like to define the scaling limit that decouple massive fields given in (3.10) and leads to a theory of massless fields together with auxiliary fields only. To do this we propose the scaling limit in the form

$$\begin{aligned} A_{11\mu}^{(L)} &= A_{11\mu}^{(L)}, & A_{12\mu}^{(L)} &= \epsilon^2 \tilde{A}_{12\mu}, & A_{21\mu}^{(L)} &= \epsilon^2 \tilde{A}_{21\mu}, \\ A_{11\mu}^{(L)} &= A_\mu - \frac{1}{2} \epsilon B_\mu, & A_\mu^{(R)} &= A_\mu + \frac{1}{2} \epsilon B_\mu, \end{aligned} \quad (4.1)$$

where ϵ is small parameter that controls the scaling limit and where we take $\epsilon \rightarrow 0$ in the end. Note also that A_μ and B_μ belong to the algebra of $\mathfrak{u}(N)$.

To begin with we insert redefined gauge fields (4.1) into the Chern-Simons Lagrangian and we obtain

$$\begin{aligned} \mathcal{L}_{CS} &= \frac{k}{2\pi} \epsilon^{\mu\nu\lambda} \text{Tr} \left(A_{11\mu}^{(L)} \partial_\nu A_{11\lambda}^{(L)} + \frac{2i}{3} A_{11\mu}^{(L)} A_{11\nu}^{(L)} A_{11\lambda}^{(L)} \right. \\ &\quad \left. + 2i\epsilon^4 A_{11\mu}^{(L)} A_{12\nu}^{(L)} A_{21\lambda}^{(L)} + 2i\epsilon^4 A_{22\mu}^{(L)} A_{21\nu}^{(L)} A_{12\lambda}^{(L)} \right) \\ &\quad + \frac{k}{2\pi} \epsilon^{\mu\nu\lambda} [-\epsilon B_\mu (\partial_\nu A_\lambda - \partial_\lambda A_\nu + i[A_\nu, A_\lambda]) + O(\epsilon^2)] \equiv \mathcal{L}^{(1)} + \mathcal{L}^{(2)}, \end{aligned} \quad (4.2)$$

where

$$\begin{aligned} \mathcal{L}^{(1)} &= \frac{k}{2\pi} \epsilon^{\mu\nu\lambda} \text{Tr} \left(A_{11\mu}^{(L)} \partial_\nu A_{11\lambda}^{(L)} + \frac{2i}{3} A_{11\mu}^{(L)} A_{11\nu}^{(L)} A_{11\lambda}^{(L)} \right), \\ \mathcal{L}^{(2)} &= -\frac{k\epsilon}{2\pi} \epsilon^{\mu\nu\lambda} \text{Tr} B_\mu (\partial_\nu A_\lambda - \partial_\lambda A_\nu + i[A_\nu, A_\lambda]). \end{aligned} \quad (4.3)$$

We see that in order to decouple the massive states we should keep k unscaled in the first part of the Lagrangian $\mathcal{L}^{(1)}$ while in the second one we should take $k = \frac{1}{\epsilon}\tilde{k}$. Then in the limit $\epsilon \rightarrow 0$ we end with

$$\begin{aligned}\mathcal{L}^{(1)} &= \frac{k}{2\pi}\epsilon^{\mu\nu\lambda}\text{Tr}\left(A_{11\mu}^{(L)}\partial_\nu A_{11\lambda}^{(L)} + \frac{2i}{3}A_{11\mu}^{(L)}A_{11\nu}^{(L)}A_{11\lambda}^{(L)}\right), \\ \mathcal{L}^{(2)} &= -\frac{\tilde{k}}{2\pi}\epsilon^{\mu\nu\lambda}\text{Tr}B_\mu F_{\nu\lambda}, \quad F_{\nu\lambda} = \partial_\nu A_\lambda - \partial_\lambda A_\nu + i[A_\nu, A_\lambda].\end{aligned}\quad (4.4)$$

Let us now give the physical interpretation of the result above. The Lagrangian density $\mathcal{L}^{(1)}$ describes $U(N-M)_k$ Chern-Simons theory living on the world-volume of fractional M2-branes that are localized at the origin of $\mathbf{C}^4/\mathbf{Z}_k$. Considering the Lagrangian density $\mathcal{L}^{(2)}$ we should split B_μ and A_μ gauge fields into U(1) and SU(N) parts as

$$\begin{aligned}A_\mu &= A_\mu^0 \mathbf{I}_{N \times N} + \tilde{A}_\mu, & \text{Tr} \tilde{A}_\mu &= 0, \\ \tilde{k}B_\mu &= \epsilon^2 B_\mu^0 \mathbf{I}_{N \times N} + \tilde{B}_\mu, & \text{Tr} \tilde{B}_\mu &= 0,\end{aligned}\quad (4.5)$$

where for later convenience we rescaled the U(1) part of B field with ϵ^2 . Then $\mathcal{L}^{(2)}$ takes the form

$$\mathcal{L}^{(2)} = -\frac{\epsilon^2 N}{2\pi}\epsilon^{\mu\nu\lambda}B_\mu^0 F_{\nu\lambda}^0 - \frac{1}{2\pi}\epsilon^{\mu\nu\lambda}\text{Tr}\tilde{B}_\mu \tilde{F}_{\nu\lambda} = -\frac{1}{2\pi}\epsilon^{\mu\nu\lambda}\text{Tr}\tilde{B}_\mu \tilde{F}_{\nu\lambda}.\quad (4.6)$$

that is precisely the gauge part of L-BLG theory.

Now we consider the scaling limit of matter fields Y^A, ψ_A . Following analysis presented in previous section we suggest that they scale as

$$\begin{aligned}Y^A &= \left(\begin{array}{c} \epsilon^2 Z^A \\ \frac{1}{\epsilon} Y_+^A \mathbf{I}_{N \times N} + \tilde{Y}^A \end{array}\right), & \text{Tr} \tilde{Y}^A &= 0 \\ \psi_A &= \left(\begin{array}{c} \epsilon \tilde{\chi}_A \\ \frac{1}{\epsilon} \psi_{A+} \mathbf{I}_{N \times N} + \tilde{\psi}_A \end{array}\right), & \text{Tr} \tilde{\psi}_A &= 0.\end{aligned}\quad (4.7)$$

Using the definition of the covariant derivative (2.4) we obtain

$$\begin{aligned}D_\mu Y^A &= \left(\begin{array}{c} \epsilon^2 \partial_\mu Z^A + i\epsilon^2 A_{11\mu}^{(L)} Z^A - Z i\epsilon^2 A_\mu^{(R)} + i\epsilon^2 A_{12\mu}^{(L)} (\frac{1}{\epsilon} Y_+^A \mathbf{I}_{N \times N} + \tilde{Y}^A) \\ \frac{1}{\epsilon} \partial_\mu Y_+^A \mathbf{I}_{N \times N} + \partial_\mu \tilde{Y}^A + i[A_\mu, \tilde{Y}^A] - iY_+^A B_\mu - \frac{i}{2}\epsilon \{B_\mu, \tilde{Y}^A\} \end{array}\right) \rightarrow \\ &= \left(\begin{array}{c} O(\epsilon)_{(M-N) \times N} \\ \frac{1}{\epsilon} \partial_\mu Y_+^A \mathbf{I}_{N \times N} + \partial_\mu \tilde{Y}^A + i[A_\mu, \tilde{Y}^A] - iY_+^A (\epsilon^2 B_\mu^0 \mathbf{I}_{N \times N} + \tilde{B}_\mu) - \frac{i}{2}\epsilon \{B_\mu, \tilde{Y}^A\} \end{array}\right) \\ &\equiv \left(\begin{array}{c} O(\epsilon)_{(M-N) \times N} \\ \frac{1}{\epsilon} \partial_\mu Y_+^A \mathbf{I}_{N \times N} + \tilde{D}_\mu \tilde{Y}^A - \frac{i}{2}\epsilon \{\tilde{B}_\mu, \tilde{Y}^A\} \end{array}\right),\end{aligned}\quad (4.8)$$

where we defined

$$\tilde{D}_\mu \tilde{Y}^A = \partial_\mu \tilde{Y}^A + i[A_\mu, \tilde{Y}^A] - iY_+^A \tilde{B}_\mu.\quad (4.9)$$

In the same way we find that

$$D_\mu Y_A^\dagger = \left(O(\epsilon)_{N \times (M-N)} \quad \frac{1}{\epsilon} \partial_\mu Y_{+A}^\dagger \mathbf{I}_{N \times N} + (\tilde{D}_\mu \tilde{Y}_A)^\dagger + \frac{i\epsilon}{2} \{\tilde{B}_\mu, \tilde{Y}_A^\dagger\}\right)\quad (4.10)$$

where

$$\left(\tilde{D}_\mu Y_A\right)^\dagger = \partial_\mu \tilde{Y}_A^\dagger - i \left[\tilde{Y}_A^\dagger, A_\mu\right] + i Y_{A+}^\dagger \tilde{B}_\mu . \quad (4.11)$$

Then using these results we find that the kinetic term for Y_A takes the form

$$\begin{aligned} \text{Tr}(D_\mu Y_A^\dagger D^\mu Y^A) &= \frac{N}{\epsilon^2} \partial_\mu Y_{A+}^\dagger \partial^\mu Y_+^A + \text{Tr} \tilde{D}_\mu \tilde{Y}_A^\dagger \tilde{D}^\mu \tilde{Y}^A - \\ &\quad - i \partial_\mu Y_{+A}^\dagger \text{Tr} \left(\tilde{B}^\mu \tilde{Y}^A\right) + i \partial_\mu Y_+^A \text{Tr} \left(\tilde{B}^{\dagger\mu} \tilde{Y}_A^\dagger\right) . \end{aligned} \quad (4.12)$$

We see that the first term diverges in the limit $\epsilon \rightarrow 0$ and hence the scaling limit in the form, as was presented in [29] seems to be not complete. The careful discussion of this issue and its resolution was given in [31] and we recommend this paper for more details. The result of the analysis presented there is that in order to have well defined scaling limit we have to add an extra term to the bosonic ABJ Lagrangian

$$N \partial_\mu U_A^\dagger \partial^\mu U^A . \quad (4.13)$$

Note that (4.13) has a "wrong" sign of the kinetic term and hence U^A can be interpreted as an extra ghost. In fact, following arguments given in [31] it is natural to extend the original ABJ action by this "ghost" term since it would be puzzling that we can derive L-BLG action that has an indefinite kinetic-term signature from a manifestly definite ABJ action by a regular scaling limit. It is also important to stress that at the level of ABJ theory the extra ghost term is decoupled. On the other hand it gets effectively coupled through the following redefinition

$$U^A = -\frac{1}{\epsilon} Y_+^A + \epsilon \frac{1}{N} Y_-^A \quad (4.14)$$

in the process when we implement the scaling limit. Note that through this redefinition we introduced new scalar field Y_-^A that plays crucial role in L-BLG theory. Then, using (4.14) we obtain that (4.12) together with (4.13) give finite contribution to the action in the limit $\epsilon \rightarrow 0$

$$\begin{aligned} &N \partial_\mu U^\dagger \partial^\mu U^A - \frac{N}{\epsilon^2} \partial_\mu Y_{A+}^\dagger \partial^\mu Y_+^A - \text{Tr} \tilde{D}_\mu \tilde{Y}_A^\dagger \tilde{D}^\mu \tilde{Y}^A + \partial_\mu Y_{+A} \text{Tr}(B^\mu \tilde{Y}^A) - i \partial_\mu Y_+^A \text{Tr}(B^{\dagger\mu} \tilde{Y}_A^\dagger) \\ &= -\partial_\mu Y_+^A \partial^\mu Y_{-A}^\dagger - \partial_\mu Y_{+A}^\dagger \partial^\mu Y_-^A - \text{Tr} \tilde{D}_\mu \tilde{Y}_A^\dagger \tilde{D}^\mu \tilde{Y}^A + i \partial_\mu Y_{+A}^\dagger \text{Tr}(B^\mu \tilde{Y}^A) - i \partial_\mu Y_+^A \text{Tr}(B^{\dagger\mu} \tilde{Y}_A^\dagger) \\ &= -2 \partial_\mu X_+^I \partial^\mu \tilde{X}_-^I - 2 \partial_\mu \tilde{X}_+^I \text{Tr}(B^\mu \tilde{X}^I) - (D_\mu \tilde{X}^I - X_+^I B_\mu) \eta^{\mu\nu} (D_\nu \tilde{X}^I - X_+^I B_\nu) \end{aligned} \quad (4.15)$$

using the relations between real scalar fields $X_\pm^I, X_\pm^{I*} = X_\pm^I, \tilde{X}^{I\dagger} = \tilde{X}^I, I = 1, \dots, 8$ and complexified scalars Y^A :

$$\begin{aligned} Y_\pm^A &= X_\pm^{2A-1} + i X_\pm^{2A}, \quad Y_{A\pm}^\dagger = X_\pm^{2A-1} - i X_\pm^{2A}, \\ \tilde{Y}^A &= -\tilde{X}^{2A} + i \tilde{X}^{2A-1}, \quad \tilde{Y}_A^\dagger = -\tilde{X}^{2A} - i \tilde{X}^{2A-1}, \end{aligned} \quad (4.16)$$

and where

$$D_\mu \tilde{X}^I = \partial_\mu \tilde{X}^I + i \left[A_\mu, \tilde{X}^I\right] . \quad (4.17)$$

Note that (4.15) takes precisely the same form as the bosonic kinetic term in L-BLG theory, up to trivial rescaling of scalar fields. Let us now consider the scaling limit of the kinetic term for fermions. If we insert (4.1), (4.7) and (4.5) into it and then take the limit $\epsilon \rightarrow 0$ we obtain

$$i\text{Tr}(\bar{\psi}^{A\dagger} \gamma^\mu D_\mu \psi_A) = iN \frac{1}{\epsilon^2} \bar{\psi}_+^{A\dagger} \gamma^\mu \partial_\mu \psi_{A+} + i\text{Tr}(\bar{\psi}^{\dagger A} \gamma^\mu \tilde{D}_\mu \tilde{\psi}_A) - i\bar{\psi}_+^{A\dagger} \gamma^\mu \text{Tr} \left\{ \tilde{B}_\mu, \tilde{\psi}_A \right\} . \quad (4.18)$$

In the same way as in the analysis of the bosonic kinetic term we add to the Lagrangian the ghost contribution

$$-iN \bar{V}^{\dagger A} \gamma^\mu \partial_\mu V_A , \quad (4.19)$$

where now V^μ is fermionic field. Then if we perform following redefinition

$$V_A = -\frac{1}{\epsilon} \psi_{+A} + \frac{\epsilon}{N} \psi_{-A} , \quad (4.20)$$

where we introduced new fermion field ψ_{-A} we find that the kinetic term for fermions (4.18) together with (4.19) are well defined even in the limit $\epsilon \rightarrow 0$

$$i\text{Tr} \bar{\psi}^{\dagger A} \gamma^\mu \tilde{D}_\mu \tilde{\psi}_A - i\bar{\psi}_+^A \gamma^\mu \text{Tr} \left\{ \tilde{B}_\mu, \tilde{\psi}_A \right\} + i\bar{\psi}_+^{A\dagger} \gamma^\mu \partial_\mu \psi_{-A} + i\bar{\psi}_-^{A\dagger} \gamma^\mu \partial_\mu \psi_{+A} . \quad (4.21)$$

Then it is easy to see that this final expression can be rewritten into the manifestly SO(8) invariant fermionic kinetic term of L-BLG model.

Let us now consider the scaling limit in the scalar terms in (2.2) and (2.3). In fact, the analysis will be almost the same as in [29] with difference that we have to explicit show that the modes Z^A decouple in the scaling limit. For example, let us consider following contribution to the bosonic potential (2.3)

$$\frac{1}{k^2} \text{Tr} \left(Y^B Y_B^\dagger Y^C Y_C^\dagger Y^A Y_A^\dagger \right) . \quad (4.22)$$

Then using the scaling limit of scalar fields (4.7) we obtain

$$\begin{aligned} & \frac{1}{k^2} \text{Tr} \left(Y^B Y_B^\dagger Y^C Y_C^\dagger Y^A Y_A^\dagger \right) \\ &= \frac{\epsilon^4}{k^2} \text{Tr} \left(\begin{array}{c} \epsilon^A Z^B Z_B \\ \epsilon Y_+^B Z_B^\dagger + \epsilon^2 Y_B^\dagger Z^B \\ \frac{1}{\epsilon^2} Y_+^B \tilde{Y}_{+B}^\dagger + \frac{1}{\epsilon} (Y_+^B \tilde{Y}_B^\dagger + Y_{+B}^\dagger \tilde{Y}^B) + \tilde{Y}^B \tilde{Y}_B^\dagger \end{array} \right)^3 \\ &= \frac{1}{\epsilon^2 k^2} \text{Tr} \left(Y_+^A Y_{+A}^\dagger + \epsilon \left(Y_+^A \tilde{Y}_A^\dagger + Y_{+A}^\dagger \tilde{Y}^A \right) + \epsilon^2 \tilde{Y}^A \tilde{Y}_A^\dagger \right)^3 + O(\epsilon^4) \end{aligned} \quad (4.23)$$

and we see that the modes Z^A really decouple. Further, the final expression takes exactly the same form as the contribution to the potential of $U(N) \times U(N)$ ABJM theory. However the analysis of this potential was performed in [29] with the following results. Due to the decomposition of the Y^A modes into trace part Y_+^A and traceless parts \tilde{Y}^A and using the fact that the potential is sextic we find that the potential is sum of $V_B = \sum_{n=0}^6 V_B^{(n)}$ where

$V_B^{(n)}$ contains n Y_+ fields and $(6 - n)$ \tilde{Y} fields. Further, using the fact that the potential is multiplied with $\frac{1}{k^2}$ we obtain that $V_B^{(n)}$ term scales as ϵ^{2-n} in the limit $\epsilon \rightarrow 0$. Then it can be shown that the terms $V_B^{(n)}$ vanish for $n > 3$. On the other hand the potential terms with $n < 2$ vanish in the limit $\epsilon \rightarrow 0$ and the non-zero contribution comes from $V_B^{(2)}$ part of the potential. Then it was further shown in [29] that this potential has full $SO(8)$ symmetry and finally that this potential exactly reproduces potential of L-BLG theory.

In the same way we can analyze the scalar terms in (2.2) that contain both fermions and bosons. Let us consider for example an expression

$$\frac{2\pi}{k} \text{Tr} \left(\bar{\psi}^{\dagger A} \psi_A Y_B^\dagger Y^B \right) . \tag{4.24}$$

Then using the scaling (4.7) and $k = \frac{1}{\epsilon} \tilde{k}$ we find that in the limit $\epsilon \rightarrow 0$ (4.24) reduces into

$$\begin{aligned} \frac{2\pi}{k} \text{Tr} \left(\bar{\psi}^{\dagger A} \psi_A Y_B^\dagger Y^B \right) &\rightarrow \frac{2\pi}{k\epsilon^3} \text{Tr} \left(\bar{\psi}_+^{\dagger A} I_{N \times N} + \epsilon \bar{\psi}^{\dagger A} \right) \\ &\times \left(\psi_{B+} I_{N \times N} + \epsilon \tilde{\psi}_B \right) \times \left(Y_{+B} I_{N \times N} + \epsilon \tilde{Y}_B \right) \left(Y_+^B I_{N \times N} + \epsilon \tilde{Y}^B \right) \end{aligned} \tag{4.25}$$

that has exactly the same form as in $U(N) \times U(N)$ ABJM theory and consequently the analysis performed in [29] can be applied for this case as well.

In summary, we have found the scaling limit of ABJ theory defined by (4.1) and (4.7) that leads to the $2 + 1$ dimensional $U(M - N)$ CS theory of level k that describes $M - N$ fractional M2-branes localized at the core of $\mathbf{C}^4/\mathbf{Z}_k$ and to $SU(N)$ L-BLG theory that describes N M2-branes infinity far from singularity. Then it is natural that these two theories are completely decoupled.

Note added: After submitting the first version of this paper to arXiv archive we were noticed by S.J. Rey about his forthcoming paper [37] that has some overlap with us.

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